GEOMETRY OF THE IMAGINARY

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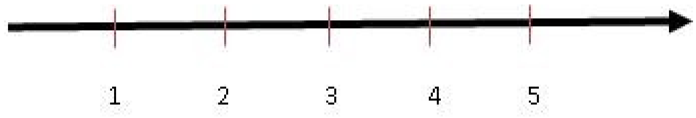
This is an attempt at discovering the reasons for the use of complex numbers in today's sciences, and to highlight the connection between the mathematics and the physics which is utilized. First of all one needs to identify the connections which are, to our experience, holding true.

First is, that the experience of counting, is valid. As far as one is able to separate two entities as distinct, all the natural numbers are hence entirely valid for physics as well as mathematics. Hence the complete theory of natural numbers can be utilized in when seeking relationships within the physical domain. Note, that this same process is also present when we divide one unit into halves... the process is the same for positive rational numbers, because they are combinations (divisions) of natural numbers.



Now, the second concept that is added with the counting, is the concept of non-counting. That is the zero point, where one does not perceive anything, or, one refuses to count! That is the "end" of the natural counting process.

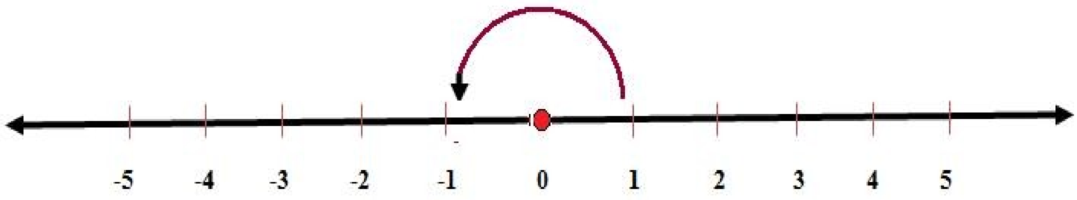
The third important concept is the scaling, where the units in which we count itself, undergoes a change. Since this change in the units is not subject to the same laws that the counting itself was, this shows a *uniform* behaviour, as opposed to the discrete behaviour observed earlier. From this we get the origin of all the motions which are uniform motions... they are not motions as we normally encounter them, but a change of scale, which is proceeding in a particular fashion.

Something different enters our treatment with the introduction of the negative numbers. Here we are counting how much we have removed when to start with there was none at all. Also, one is introducing a dichotomy here for the first time. You can not only have numbers which have magnitude, but they have another quality which enables them to be removed from existence. Since negative quantities are never encountered in ordinary physical phenomena, we have adopted the method of shifting the zero or reference point, in order to count "backwards". Here, the physical world failed to conform immediately to our mathematical expectations, so we adjusted the application of the concepts so that we can still use it. Technically speaking, shifting the zero point means calculating increases and decreases from a previously determined count. This is the start of the expression of the reciprocal relation between space and time, where an increase in time is the same as the decrease in space.

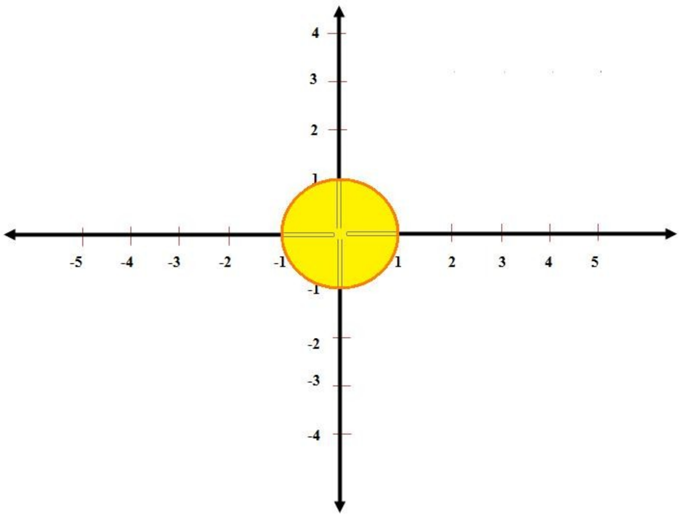
Along with the counting "backwards" concept, we notice that the "backwards" implies a direction, where none existed before. Thus, a reduction in a physical quantity from null, was not observed. The mathematics however was equally applicable to decreases from a given quantity, as well as to a change of direction. The concept of angle is derived from the fact that we chose a reference point, and then a reference direction. Hence, angles come into being because we choose a reference point.

The actual behavior of negative quantities, however, will not occur in any domain where only one of the two aspects are changing. That is, changing space alone or time alone will not suffice for negative quantities, in one dimension, as we know the terms in reference to the Reciprocal System.
The appearance of negative quantities occurs only when more than one dimension is involved. And it must be carefully noted that there two types of negative quantities, those with a reference point dependence (can be in any number of dimensions) and those which are *actually* negative, meaning

they remove something from a true "zero". So, the negative number which we use ubiquitously is just the first of the two possibilities.



Having made that clear, we can now examine what happens when we increase the number of dimensions and take a look at the motions involved. First of all, an increase from one to two dimensions helps us to look at the circle, which is a two dimensional entity, and hence circular motion.



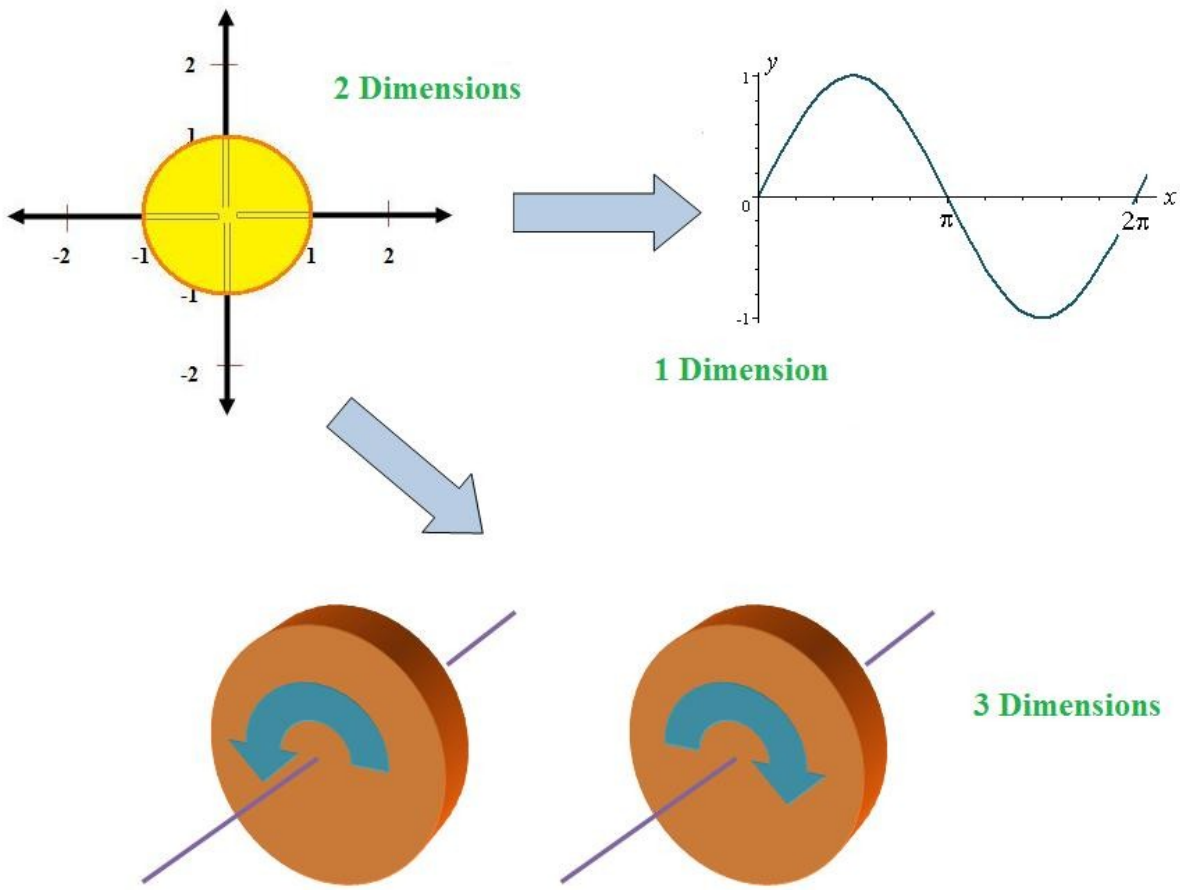
We must now examine what is happening when we start *transformations* of the motion. These transformations can be generalized as projections, and we will use some concepts from projective geometry as well here. A motion in say 2 dimensions can be projected onto:

1. Another motion in *1* dimension.
2. Another motion in *2* dimensions.
3. Another motion in *3* dimensions.

In other words, projections can be done across dimensions, which gives rise to different geometries, as we shall soon see. If we project a motion, say a circular motion in a plane which is an acceleration, onto another motion in the plane, it merely amounts to a rescaling of the geometry... it is a metric transformation. However, when we transform the same onto a single dimension, the line, from a uniform motion, we get the of a sine curve, or a cosine, but with *real* coordinates. However, this coordinate now has a *varying* acceleration.

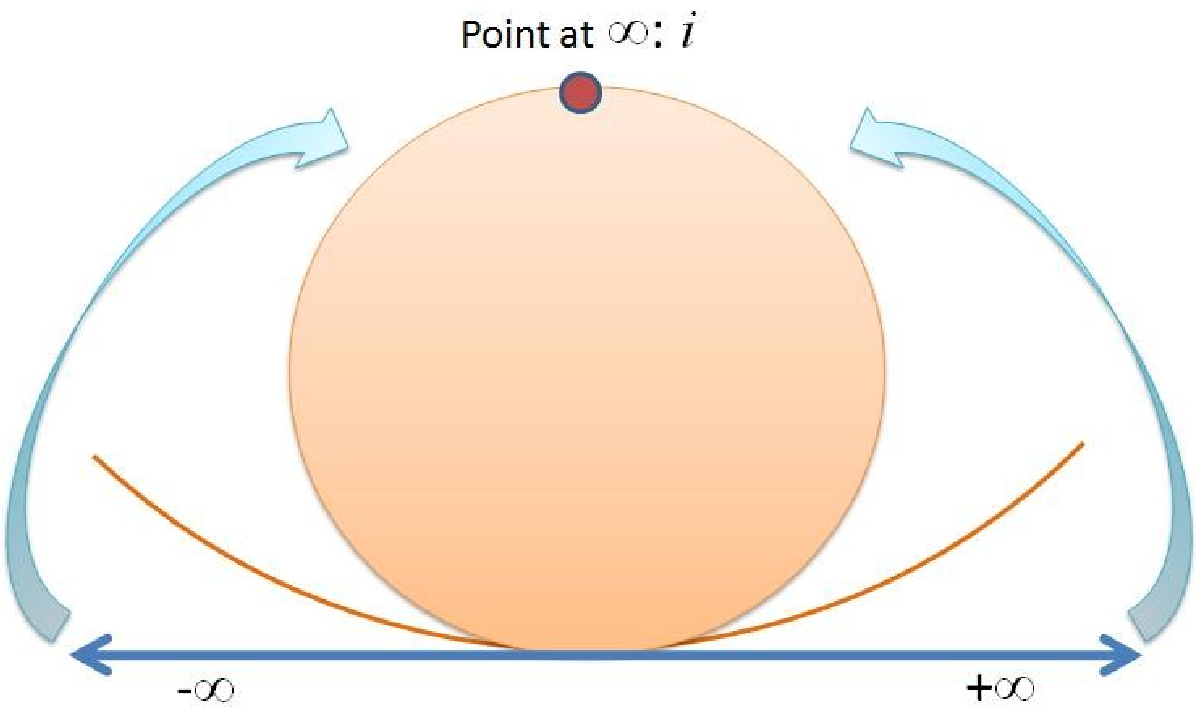
We can also project the motion onto three dimensions, in which case we transform a polarity which was there earlier: Increase and decrease in angle into clockwise and anti-clockwise motion. It is important to note that in two dimensions alone, there was no way of converting a clockwise motion into an anti-clockwise motion, nor of identifying whether it is clockwise or anticlockwise in the first place, but in three dimensions, we can change our reference point along the third axis, or flip the motion around the third axis, and get both clockwise AND anti-clockwise motion from the same motion. "Clockwise" has meaning only from the third dimension.

This gives us two "rules of thumb", as it were: when projecting into lesser dimensions, something which was uniform becomes time varying. When projecting into a higher dimension, something that had polarity, now becomes degenerate, where the two polarities can be seen to be equivalent to a sign change made possible by the extra dimension.



Let us now apply the same logic to an essentially one dimensional motion: a uniform straight line motion, at constant velocity.

Its projection onto zero dimensions, a point, would be just that, a single point, going "blip" in the time variation. However, the more interesting case is when we try to project it on to a two dimensional region. How do we do that? We need to consider a concept from projective geometry, which essentially does the same as what happened in the two dimensional circular motion... it creates a polarity, and also gives a way to resolve it. The polarity in this case is whether it goes left or right. In either case, it still goes through the *same* point at infinity (it is both positive and negative infinity), which is an "imaginary" point. Now, in order to make our line into a two dimensional entity via projection, we have to consider what happens to a circle whose radius is slowly taken to infinity: it looks more and more like a straight line. In this case, we apply the the exact reverse, we bring the point at infinity towards the line, hence curving it more and making it a circle, whose one point is imaginary, and is hence an imaginary circle, and two dimensional as we required it. That is the origin of imaginary numbers, they come into being when we project a motion of lesser dimensions onto higher dimensions, in which case we will have to bring the point at infinity into our reckoning. Real line + point at infinity: imaginary circle. The imaginary number is the number that directs one towards the point at infinity.



The second consequence is that the new dimension, since it is added afresh, has to be independent of reference points, and hence needs to be an absolute scale such as the natural reference system (1/n, 1, n). Hence, the polarity has to be multiplicative, and also it does not come into being till we FIX a reference point either and hence the scale is ($1/i$, 0, i). The number i that is used has all the properties necessary, with i itself remaining beyond measurement. Now, $i^2 = -1$, where the negative sign is used in the meaning described earlier. The square shows that if I go all the way to zero to infinity (i) and then cross it and then come back to the line again (another i) I would have done the same as approaching the line from negative infinity to zero. Thus the sign has to flip from positive to negative infinity, which is what is done by i^2. The sign of the point at infinity is not determinate, as it is neither positive nor negative, but provides a route to change the sign. That IS the imaginary number's defining quality.

With that in hand, we can now understand the similarities and differences of the sines and cosines arising from the "real circle" and the "imaginary circle". The real circle is two dimensional, and hence the projection of the motion onto a single dimension generates the sine or the cosine. Here both the circular motion *and* the linear motion are hence measurable, as we are lowering the dimensions while projecting. Now, when we project linear motion onto a circular motion, by raising dimensions, either the sine or the cosine is measured, while the other is imaginary. Thus, in all cases in physics where this is done, for example in electrical engineering where motion in a line (current) is made into a "circuit", which is actually supposed to be two dimensional, a one dimensional quantity gets to be represented using imaginary numbers. So far the general physical explanation has been something non-scientific: "convenience". We use complex numbers in them because it is convenient, but there is a reason for that convenience, and this highlights it. Hence the confusion as to whether the imaginary number means something real or not can now be resolved.

Something that also makes sense from this is to see how "dimensional reduction" works in the reciprocal system... here the "imaginary" components cancel out. Hence, it is just a special instance where the projection actually *equals* the motion.